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Friction on a Spinning Piece of Matter

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In our Universe rotating motion, contrary to translational motion, is defined absolutely. I examine some physical consequences of the existence of absolute rotation. One of them is a resistive torque on a rotating piece of dielectric induced by the scattering of the fluctuations of the QED vacuum. This torque rests on the estimation of the emission of angular momentum by a wave scattered by a rotating dipole. This phenomenon *per se* could also yield a method for cooling the rotational degrees of freedom of molecules.

KEY WORDS: Rotation; Casimir effect; rotational cooling.

One of the simplest dynamical systems is a piece of matter rotating at constant angular speed. Surely its image was in the minds of those who began to think about the various possible motions to emerge in the world, be it at large scales in the Ptolemaic explanation of the trajectories of celestial bodies or in the oscillations of a pendulum. To understand the oscillations of a pendulum Newton did not use sine or cosine functions, but rather their kinematic representation by projecting onto the line circular motion at constant speed. In this homage to Mitch Feigenbaum, I present some variations on the physics of the rotating motion, even though the underlying dynamics are far more trivial than the beautiful and fascinating structures Mitch Feigenbaum uncovered in his famous work.

Newton in his particularly well thought out introduction to the Principia makes a very thorough distinction between translation at constant speed, which cannot be detected *in vacuo* in a comoving frame and rotation at constant angular speed, which can be detected[8] by looking at the curvature of the surface of a liquid at rest in its rotating frame. This deep

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remark made me think about other possibilities of detecting absolute rotation and possible consequences of it.

The physical world is such that absolute rotation exists. Therefore one may expect that a rotating object should come back to rest spontaneously because it makes an excited state that should decay – in the quantum sense – to the ground state. This depends of course on the full system: the rotating piece of matter and the rest of the Universe make a single system by their interaction. Consider a rotating piece of dielectric interacting with the 'Universe' seen as the QED (quantum electrodynamic) vacuum at zero temperature (see remarks at the end on the finite temperature case). The natural question to ask is: Does the interaction between the QED vacuum and the rotating dielectric yield some sort of friction, a kind of irreversible Casimir-like force? The existence or not of such 'quantum braking' – to use the same terminology as in ref. 2 – relies on a rather long chain of arguments that I shall summarize below. A first difficulty with this idea is that it seems to show a direction of time, in apparent contradiction with the time reversal symmetry of the laws of physics. The link between macroscopic irreversibility and microscopic reversibility is a difficult question, and I shall try to limit myself to the simplest possible considerations.

This paper is organized as follows. I derive first the formula of the torque radiated by a time dependent electric dipole. Indeed there is nothing new there and the final result can be found in Landau and Lifshitz's classical theory of fields.[3] However Landau's expression is related implicitly at least to the torque generated by the emission of an EM (electromagnetic) wave by a rotating permanent dipole. Therefore I shall detail this calculation for the scattering of an EM wave by a rotating dielectric, a physical situation different of the one of a rotating permanent dipole and which does not seem to have been considered before. Then I use this formula of the radiated torque to evaluate the damping by the scattering of the fluctuations of the QED vacuum. This requires some rather crude approximations, in particular because the starting point is purely classical (non-quantum) although the final result is a purely quantum effect. This is in line with estimates of the order of magnitude of the Casimir effect which can be done classically until the very end when the magnitude of the fluctuations of the EM field is found by using the rules of quantum mechanics.[6] Then I consider another physical situation where this phenomenon of radiation of torque is relevant, namely what happens when the incident EM field is a classical light beam. This gives a way, at least theoretically, for cooling rotating objects by radiating their angular momentum. Indeed, if the object under consideration is a molecule, taking it as classical rigid rotator with polarization coefficients is not fully justified because one should take into account quantum selection rules for the interaction between the radiation and the molecule. I plan to come back to this question in a future publication.

The starting point, as for any Casimir-like effect, is to compute the resistive torque exerted on a rotating piece of dielectric for each individual mode of the classical EM field. This torque is proportional to the square of the amplitude of the electric field; the rules of quantum mechanics are such that the average value of this square is not zero in the ground state or 'QED vacuum'. This first step relies on the properties of the scattering of an incident EM wave by the rotating dielectric. The sought after effect is the flux of angular momentum in the scattered wave field; this flux, if it exists, has to be balanced by a torque on the rotating scatterer. This torque is derived from the balance of angular momentum. Physically it would come from the interaction between the radiated wave and the rotating dipole. But this step is no more necessary there than when computing the Lorentz–Dirac friction on an radiating dipole. The force, as the torque, is derived from what is radiated at long distance by using the general property of balance of linear and/or angular momentum. Consider an EM wave incident on a rotating dielectric. The calculation of the friction torque is inspired by a calculation(7) of the friction force by the zero-point fluctuations in quantum condensed state at zero temperature.[1] The zero-point fluctuations of the EM field have on average all possible symmetry properties associated with the Lorentz and rotational invariance. These vacuum fluctuations form the incoming state before scattering by the rotating dipole. On average the vacuum fluctuations have no angular momentum before scattering, but they do after. Since the scattering problem must be solved in a non-rotating frame, the scatterer is time dependent too. Therefore, if the angular frequency of the incident wave is ω, after scattering it becomes $\omega \pm n\Omega$, where Ω is the angular speed of rotation and n a positive integer.

The flux of angular momentum far from the scatterer is derived by integration of the torque generated by the Maxwell tensor. I shall rely on results and notations of Jackson's book on classical electromagnetism,(1) quoted simply as 'Jackson' later on. The starting point of this calculation of the radiated torque is the formula (9.18) in Jackson (Eq. (2) below). This yields the electric field of an electric dipole depending periodically on time without any approximation. Let $\mathbf{P}(t)$ be this dipole, boldface being for vectors. Its time-dependent behaviour is assumed to be harmonic, like

[1] In quantum condensed phases the zero-point fluctuations are usually called quantum depletion. They represent a small effect in Bose–Einstein condensates in dilute vapours, but 80% of the mass in the ground state of superfluid liquid Helium.

$$\mathbf{P}(t) = \frac{1}{2}\left(\hat{\mathbf{P}} e^{i\omega t} + \hat{\mathbf{P}}^* e^{-i\omega t}\right). \tag{1}$$

This assumes a single Fourier component. As usual, the quantity denoted as $\hat{\mathbf{P}}$ is a vector with complex-valued components, not time-dependent, and $\hat{\mathbf{P}}^*$ is its complex conjugate. The quantities multiplying complex exponential as $e^{i\omega t}$ will be denoted with a hat. Quantities without hat are real and have a direct physical meaning, their relation to the hatted quantities being as in (1). Actually I shall need more than one Fourier components, but this does not have to be taken into account explicitly at this stage. The EM field obeys the Maxwell equations, and it has the wavenumber $k = \frac{\omega}{c}$, c being the speed of light. Furthermore let the dipole be located at $r = 0$. According to Jackson (equation 9.18) the electric field of this dipole is:

$$\hat{\mathbf{E}}_{\text{dip}} = k^2 \frac{e^{ikr}}{r}\left[\hat{\mathbf{P}} - \mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{P}})\right] + \left[3\mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{P}}) - \hat{\mathbf{P}}\right]\left(\frac{1}{r^3} - \frac{ik}{r^2}\right)e^{ikr}, \tag{2}$$

where $\mathbf{n} = \frac{\mathbf{r}}{r}$. This neat expression includes the familiar field of a static dipole which behaves like $\frac{1}{r^3}$. The other formula in Jackson I'll need is the flux of angular momentum at infinity from the dipole. This flux depends on the term which behaves like r^{-2} in the large distance expansion of the right hand side of Eq. (2). On contrary the terms entering the various cross sections and that depend on the scattering amplitude depend on the coefficient of the first term, behaving like r^{-1} at large r. Let T_{ij} be the (ij) component of the Maxwell stress tensor, i and j referring to Cartesian coordinates. This tensor is related to the EM field by the formula:

$$T_{ij} = \frac{1}{4\pi}\left[\epsilon E_i E_j + \mu H_i H_j - \frac{1}{2}\delta_{ij}\left(\epsilon E^2 + \mu H^2\right)\right]. \tag{3}$$

In this expression, E_i is the i-component of the electric field and H_j the j-component of the magnetic field. Moreover δ_{ij} is the Kronecker delta, equal to 1 if $i = j$ and to zero otherwise. Finally ϵ is the electric permittivity of the medium (here the vacuum) and μ its magnetic permittivity. According to the results of Exercise 6.10 in Jackson, the flux of angular momentum $\mathbf{Q}$ is a vector such that its component j has the expression:

$$Q_j = e_{jkl} n_i T_{ik} r_l,$$

with the convention of summation over the repeated indices. The definition of Q_j implies three summations, over the indices i, k and l, e_{jkl} being the fully antissymmetric Levi–Civita tensor. The physical meaning of $\mathbf{Q}$ is that by mutiplying it by the element of area of a surface with unit normal $\mathbf{n}$ one gets the total flux[2] of angular momentum across this surface. Therefore the total loss (or gain) of angular momentum by radiation per unit time is obtained by integrating this flux all over a sphere surrounding the dipole. Using the expression of the stress tensor in Eq. (3), one obtains for $\mathbf{Q}$:

$$\mathbf{Q} = \frac{\epsilon}{4\pi}(\mathbf{n}\cdot\hat{\mathbf{E}}_{\text{dip}})\hat{\mathbf{E}}^*_{\text{dip}} \times \mathbf{r}. \tag{4}$$

In Eq. (4), the symbol $\times$ denotes the vector product. The contribution of the magnetic field has been neglected, a realistic approximation in most cases, where the magnetic contribution to scattering is negligible compared to the one of the electric field. Furthermore $\hat{\mathbf{E}}^*_{\text{dip}}$ is the complex conjugate of $\hat{\mathbf{E}}_{\text{dip}}$ assuming that the physical electric field $\mathbf{E}$ is a sum of periodic functions of time as $\frac{1}{2}\left(\hat{\mathbf{E}}e^{i\omega t} + \hat{\mathbf{E}}^* e^{-i\omega t}\right)$. Each Fourier component of $\mathbf{E}_{\text{dip}}$ yields a constant (in time) contribution to $\mathbf{Q}$ that is given by the Eq. (4), a sum over all frequencies being implied. From Eq. (2), the vector product $\hat{\mathbf{E}}^*_{\text{dip}} \times \mathbf{r}$ has the following expression for each frequency component of the scattered field:

$$\hat{\mathbf{E}}^*_{\text{dip}} \times \mathbf{r} = e^{-ikr}\left(\frac{k^2}{r} - \frac{1}{r^3} - \frac{ik}{r^2}\right)\hat{\mathbf{P}}^* \times \mathbf{n}.$$

The scalar product $\mathbf{n}\cdot\hat{\mathbf{E}}_{\text{dip}}(\mathbf{n} = \frac{\mathbf{r}}{r})$ has the value:

$$\mathbf{n}\cdot\hat{\mathbf{E}}_{\text{dip}} = 2(\mathbf{r}\cdot\hat{\mathbf{P}})e^{ikr}\left(\frac{1}{r^3} - \frac{ik}{r^2}\right).$$

Combining the two, one gets the following equation for $\mathbf{Q}$:

$$\mathbf{Q} = 2k^2\left[\frac{ik}{r^2}\hat{\mathbf{P}} \times \mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{P}}^*) - \frac{ik}{r^2}\hat{\mathbf{P}}^* \times \mathbf{n}(\mathbf{n}\cdot\hat{\mathbf{P}})\right]. \tag{5}$$

A significant remark is that the part of $\mathbf{Q}$ that is independent of time depends on r as $\frac{1}{r^2}$. Therefore the total flux across a large surrounding sphere is constant, independent of the radius of the sphere r as

[2]Rigorously speaking, $\mathbf{Q}$ is not a flux *per se*, since the flux of a vector should be a tensor independent on $\mathbf{n}$. $\mathbf{Q}$ is a flux in the direction of $\mathbf{n}$. I shall not use this rather cumbersome wording but rather say that $\mathbf{Q}$ is the flux of angular momentum.

expected. This radiated angular momentum is the average value on the sphere (namely over all possible orientations of $\mathbf{n}$) of $\mathbf{Q}$. This average value is

$$\langle \mathbf{Q} \rangle = \frac{4ik^3}{3r^2} \hat{\mathbf{P}} \times \hat{\mathbf{P}}^*.$$

The total flux across the large sphere (a torque) is $4\pi r^2$ times this average, namely[3]:

$$\Gamma = \frac{16i\pi k^3}{3} \hat{\mathbf{P}} \times \hat{\mathbf{P}}^*. \tag{6}$$

The Eq. (6) shows that a dipole that stays oriented in the same direction does not emit any angular momentum, because then $\hat{\mathbf{P}}$ and $\hat{\mathbf{P}}^*$ are parallel and $\hat{\mathbf{P}} \times \hat{\mathbf{P}}^*$ is zero, a result to be expected. This shows too, indirectly at least, the need for a rotation of the dipole to radiate angular momentum. The last step in this calculation is to compute the vector product $\hat{\mathbf{P}} \times \hat{\mathbf{P}}^*$ when the dipole strength comes from the response of the rotating dielectric to an incident plane EM wave.

Let us write the general expression of the electric dipole $\mathbf{P}$ induced in the linear approximation by an external electric field $\mathbf{E}$ when the wavelength of the EM field is much larger than the size of the piece of dielectric. This last approximation makes the calculation tractable. Otherwise it would require either a numerical approach or a Born-like approximation. For an arbitrary orientation of the dielectric, the linear relation reads in coordinate form:

$$P_i = A_{ij} E_j. \tag{7}$$

In this equation, A_{ij} is a rank two tensor that is attached to the dielectric (it has the physical dimension of a volume with our units). If this dielectric rotates, one must rotate A as well. For an ellipsoidal dielectric of uniform permittivity, the equations for the electric field and the polarization can be solved explicitly.(5) Let (X, Y, Z) be the coordinate system attached to the ellipsoid, such that the Cartesian equation of its surface is

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} + \frac{Z^2}{c^2} = 1$$

[3] A closely related expression is in the equation (9.98) of ref. 3.

In this coordinate system the dipole induced by an uniform external field $\mathbf{E}$ has the X component:

$$P_X = A_{XX} E_X, \tag{8}$$

where

$$A_{XX} = \frac{abc}{3\left[\frac{\epsilon}{\epsilon_1 - \epsilon} + m_{(X)}\right]},$$

ϵ_1 is the permittivity of the piece of dielectric and ϵ the one of the outside medium (practically the vacuum there). Furthermore, the quantity called $m_{(X)}$ is, up to a multiplicative constant, an elliptic integral:

$$m_{(X)} = \frac{abc}{2} \int_0^\infty \frac{d\zeta}{(\zeta + a^2)R(\zeta)},$$

where $R(\zeta) = \sqrt{(\zeta + a^2)(\zeta + b^2)(\zeta + c^2)}$. In this frame of reference, the polarization tensor is diagonal (but not proportional to the identity!), and any other component of $\mathbf{P}$ can be derived from Eq. (8) by index permutation. To make things simpler, I consider the case of a piece of dielectric with an axis of rotational symmetry, the Z-axis, along the unit vector $\mathbf{N}$. This unit vector is attached to the dielectric and, in general, the rank two tensor A_{ij} is the sum of a tensor $\alpha N_i N_j$ and an isotropic part $\beta \delta_{ij}$, where α and β are two scalars (independent on the orientation of the dielectric). For this symmetric ellipsoid α and β can be found by considering first an electric field perpendicular to the axis of symmetry, that gives $\beta = A_{XX} = A_{YY}$ and then a field along Z,that gives $\alpha = A_{ZZ} - \frac{1}{2}(A_{XX} + A_{YY})$. The source of time dependence in the polarization is twofold: first it comes from the time-dependent electric field $\mathbf{E}$, then from the rotation of the dielectric through the rotation of the unit vector N. This is made obvious from the expression of the polarization valid in the axissymetric case:

$$\mathbf{P} = \alpha(\mathbf{N} \cdot \mathbf{E})\mathbf{N} + \beta \mathbf{E}. \tag{9}$$

I shall no longer consider the last term which comes from the isotropic part of the polarization tensor, since it does not contribute to the torque. The rotation is with the angular velocity Ω around the z-axis in the fixed coordinate system. There are two more vectors involved: the vector defining the axis of polarization of the electric field in the incident wave, assumed to be linearly polarized, and the unit vector **N**, the latter being a function of time. The case of a perfect conductor, although physically very different, would yield too a relation between the external field and the induced dipole formally identical to the one given in Eq. (9) for an axissymetric piece of dielectric.

I continue the calculation in the fixed frame of reference where the coordinates are (x, y, z). Let θ be the constant angle between the rotation axis and **N**, and let $E_x \sin(\omega t)$ and $E_z \sin(\omega t)$ be the Cartesian components of **E** in the fixed coordinate system. The time-independent vector of components E_x, E_y will be denoted as $\tilde{\mathbf{E}}$. It represents the electric field of the incoming wave, but for the sine factor $\sin(\omega t)$. The Cartesian components of **N** are

$$N_x = \sin(\theta)\cos(\Omega t), \tag{10}$$

$$N_y = \sin(\theta)\sin(\Omega t), \tag{11}$$

and

$$N_z = \cos(\theta). \tag{12}$$

The Cartesian components of the polarization **P** are:

$$\begin{aligned} P_x = \alpha\,(\mathbf{N}\cdot\mathbf{E})\,N_x &= \alpha\sin(\omega t)\sin(\theta)\sin(\Omega t) \\ &\quad\times[E_x\sin(\theta)\sin(\Omega t) + E_z\cos(\theta)]. \end{aligned} \tag{13}$$

This is equivalent to

$$\begin{aligned} P_x &= \frac{\alpha}{4}E_x\sin^2(\theta)\,[2\sin(\omega t) - \sin(\omega+2\Omega)t - \sin(\omega-2\Omega)t] \\ &\quad+\frac{\alpha}{2}E_z\sin(\theta)\cos(\theta)\,[\cos(\omega-\Omega)t - \cos(\omega+\Omega)t]. \end{aligned} \tag{14}$$

There is a parallel expression for the y-component of **P**:

$$\begin{aligned} P_y = \alpha\,(\mathbf{N}\cdot\mathbf{E})\,N_y &= \alpha\sin(\omega t)\sin(\theta)\cos(\Omega t) \\ &\quad\times[E_x\sin(\theta)\sin(\Omega t) + E_z\cos(\theta)]. \end{aligned} \tag{15}$$

This can also be written as

$$\begin{aligned} P_y &= \frac{\alpha}{4} E_x \sin^2(\theta)\,[\cos(\omega - 2\Omega)t - \cos(\omega + 2\Omega)t] \\ &\quad + \frac{\alpha}{4} E_z \sin(\theta)\cos(\theta)\,[\sin(\omega + \Omega)t + \sin(\omega + \Omega)t]\,. \end{aligned}$$

The z-component of **P** is written below for the sake of completeness, although it does not contribute to the torque:

$$\begin{aligned} P_z = \alpha\,(\mathbf{N}\cdot\mathbf{E})\,N_z &= \alpha \sin(\omega t)\cos(\theta) \\ &\quad \times [E_x \sin(\theta)\sin(\Omega t) + E_z \cos(\theta)]\,. \end{aligned} \tag{16}$$

The shifted frequencies $\omega \pm \Omega$ and $\omega \pm 2\Omega$ already show-up in the expressions for P_x and P_y. This shows the change of frequency due to the rotation of the dipole. Now one has to insert those expressions of the components of the polarization vector into the radiated torque given in Eq. (6). This requires that all time dependent quantities be written as sum of complex exponentials, and then to use the Eq. (6). The only non-zero component of the resistive torque is along z, the axis of rotation. It is

$$\Gamma_z = \frac{16 i\pi k^3}{3}\left(\hat{P}_x \hat{P}_y^* - \hat{P}_y \hat{P}_x^*\right).$$

In this expression of Γ_z one implicitly includes a sum over all possible Fourier components, each one with a frequency-dependent prefactor $\frac{16i\pi k^3}{3}$. Four different frequencies are involved in the final result: $\omega - 2\Omega$, $\omega - \Omega$, $\omega + \Omega$ and $\omega + 2\Omega$. The identity $\hat{P}_x \hat{P}_y^* - \hat{P}_y \hat{P}_x^* = \frac{i}{2}\left(P_{c,x}P_{s,y} - P_{s,x}P_{c,y}\right)$ make the calculation slightly easier, where $P_{s,x}$ is the real coefficient of the sine function like $\sin(\omega+\Omega)t$ in P_x, etc. The final result is the following expression for the resistive torque along the axis of rotation:

$$\begin{aligned} \Gamma_z &= \alpha^2 \frac{E_x^2}{6c^3} \sin^4(\theta)\left[(\omega - 2\Omega)^3 - (\omega + 2\Omega)^3\right] \\ &\quad + \alpha^2 \frac{2E_z^2}{3c^3} \sin^2(\theta)\cos^2(\theta)\left[(\omega - \Omega)^3 - (\omega + \Omega)^3\right]. \end{aligned} \tag{17}$$

This equation shows that, without rotation, there is no torque: Γ_z is equal to zero if Ω is zero. More generally, the resistive torque is an odd function of Ω. In the limit $\Omega \ll \omega$, Γ_z has the following limit form[4]:

$$\Gamma_z = -\frac{2\alpha^2 E_x^2}{c^3}\sin^4(\theta)\omega^2\Omega - \frac{4\alpha^2 E_z^2}{c^3}\sin^2(\theta)\cos^2(\theta)\omega^2\Omega. \tag{18}$$

The coefficient of polarization α has been taken as independent on the frequency. Such a dependence could be taken into account at least formally. The torque given in Eq. (18) is proportional to the square of the incoming electric field. Therefore one may expect that such a torque would come from the scattering of the quantum fluctuations of the QED vacuum: the mean value of a quantity proportional to the square of a quantum fluctuation is not zero in general, as the existence of the Casimir effect clearly shows. The order of magnitude of this effect is obtained by adding the contributions of all modes of fluctuation of the QED vacuum, seen here as defining the amplitude of an incoming wave. I shall not try to make a detailed calculation of this effect, and give order of magnitude estimates only. The contribution of a mode of frequency ω to $|E|^2$ is of order $\frac{\hbar\omega}{2V}$, where V is the volume of the system. In the frequency interval $[\omega, \omega + d\omega]$ the number of modes of the EM field is $\frac{8\pi V\omega^2 d\omega}{c^3}$. Therefore in the limit Ω small the sum of all contributions to the resistive torque is $\Gamma_C \sim \Omega \int_0^\infty d\omega\, \alpha^2 \frac{\hbar\omega^5}{2c^3}$ where the subscript C is for 'Casimir'. Taking α to be a constant makes this expression diverge massively in the large frequency/short wavelength domain. Two physical length scales are there to cut-off this divergence at short distance: first the polarization α tends to zero as the wavelength tends to zero. The typical length scale involved is the wavelength of the EM waves at frequencies of the order of the plasma frequency of the material, with the electron density taken with all the electrons in this material. This is a very short wavelength at the usual densities in condensed matter, typically in the UV or even X-ray range. In the present problem, where I assumed that the rotating piece of dielectric is much bigger than the wavelength of the incoming EM wave, this is not the relevant short range cut-off. The physical cut-off comes from this assumption: if the wavelength of the incoming EM wave is of the order of or shorter than the size of the dielectric one cannot assume anymore a simple relation like in (9) between the incoming field and the polarization. Therefore, at least as far as the domain of validity of the present theory is concerned, one must limit its application to wavelength that are larger than or of the

[4]This expression could be written in a geometrically covariant form by using dot products of the three vectors involved, namely $\mathbf{N}$, $\tilde{\mathbf{E}}$ and Ω, by writing $E_x^2 = \tilde{\mathbf{E}}^2 - \frac{(\Omega\cdot\tilde{\mathbf{E}})^2}{\Omega^2}$, $E_z^2 = \frac{(\Omega\cdot\tilde{\mathbf{E}})^2}{\Omega^2}$ and $\cos^2(\theta) = \frac{(\Omega\cdot\mathbf{N})^2}{\Omega^2}$. However the final result is not obviously simpler than Eq. (18).

same order as the size of the rotating dielectric. The approximate value of the resistive torque Γ_C due to the long wave fluctuations of the QED vacuum is found by limiting the diverging integral to waves with a wavelength larger than or on the same order as the length of the rotating dielectric: $|\omega| \leqslant \frac{2\pi c}{l}$, l being the length of this piece of dielectric. Assuming furthermore that it has a finite aspect ratio, one finds that $\alpha \sim l^3$. Putting all these estimates into the expression of the resistive torque, one gets the very simple estimate $\Gamma_C \sim \hbar\Omega$, quite a large effect even for objects noticeably bigger than a molecule.

Another possible consequence of this resistive torque could be the damping of the rotation of molecules with a rod-like or disc-like shape. By shining a light beam on the molecule one would bring its rotation to rest. This could be a way of cooling a molecular gas, provided the cooling of the rotational level by this process is efficient enough to overbalance any heating of the vibrational and/or translational degrees of freedom by the same light beam. There should also be enough exchange with the other degrees of freedom to make this cooling efficient for all of them. Let us estimate the order of magnitude of the efficiency of this cooling. The order of magnitude of the resistive torque is $\Omega\frac{\omega^2 d^2}{c^3}$, where d is the magnitude of the electric dipole induced by the external field. If this resistive torque slows down a rotating molecule of moment of inertia I, the typical decay time for the rotation will be $\tau \sim \frac{Ic^3}{\omega^2 d^2}$. One can check the coherence of this order of magnitude estimate, since d^2 has the physical dimension of an energy times a length cube. To estimate d, the magnitude of the dipole induced by the incoming EM wave, I notice that an electric field on the order of E_0 – one volt per Angstrom – would yield an electric dipole er_{B}, where e is the charge of the electron and r_{B} the Bohr radius, E_0 being of course the electric field inside a Bohr atom. Assuming now that the electric dipole is proportional to the incoming field, one obtains $d \sim er_{\mathrm{B}}\frac{E}{E_0}$. The order of magnitude of the moment of inertia I is Mr_{B}^2, M being the mass of the molecule, much larger than the mass of the electron. Introducing the electron mass explicitly together with the fine stucture constant $\alpha_e = \frac{e^2}{\hbar c}$, one obtains $\tau \sim \frac{M}{m}\frac{\alpha_e}{r_{\mathrm{B}}}\frac{c}{\omega^2}\left(\frac{E_0}{E}\right)^2$. The combination $\frac{\alpha_e}{r_{\mathrm{B}}}\frac{c}{\omega^2}$ is the frequency of the Bohr atom, ω_{B}. Combining all this one obtains the compact expression:

$$\tau \sim \frac{M}{m}\frac{\omega_{\mathrm{B}}}{\omega^2}\left(\frac{E_0}{E}\right)^2.$$

To give an idea of the order of magnitude of this relaxation time, one can take $\omega \sim \omega_{\mathrm{B}}$. This amounts to a period of the incident wave in the range of 10^{-14} s. The final value is very sensitive to the value of E. An electric field of 100 V/cm, that does not seem to be unrealistic, yields $\left(\frac{E_0}{E}\right)^2 \approx 10^{12}$ and finally τ in the 10–100 s range. This estimate could be refined in a specific example. In particular the magnitude of the induced dipole could be increased by taking an incoming light close to an absorption line of the molecule. As far as I am aware this method of cooling rotational degrees of freedom has not been proposed in the literature. It is not even clear that this resistive torque has ever been considered, at least for the case of a dipole induced by scattering, instead of the more obvious case of a rotating permanent dipole.

To conclude I address a rather intricate question. From the calculation presented, one could get the (wrong) impression that the interaction between an EM wave and a dipole always leads to decay of the rotational degrees of freedom of this dipole. Indeed any wave incident on the dipole can be decomposed into a sum of plane waves and, by scattering, each plane wave seems to take out some angular momentum. In particular this would rule out the relaxation to thermal equilibrium of a dipole interacting with black-body radiation at a finite temperature: if one assumes that the only degree of freedom of this dipole is rotational, and if scattering always takes out angular momentum, one would have realized an anti-Carnot engine, putting the energy of the rotator irreversibly into a single thermal bath. The way out of this apparent paradox is that the radiation field can have incoming and outgoing angular momentum. There is some incoming angular momentum whenever the EM wave is circularly polarized. Of course in black-body radiation this incident wave is circularly polarized at random, fluctuating constantly from one polarization sign to the other. Nevertheless, there is an instantaneous state of circular polarization. Suppose the dipole is a rest (it does not rotate), and that it is submitted to an incoming circularly polarized wave. This wave will induce a polarization of the dipole. By an effect similar to the one discussed in reference,[4] the retardation between the polarization and the polarizing field will be such that the cross product $\mathbf{P} \times \mathbf{E}$ (that is the torque on the dipole) is not zero on average. This yields a non-zero constant torque due to an incident circularly polarized wave. For black-body radiation this yields a fluctuating torque. If the frequency of the average radiation in the black-body radiation is far higher than the inverse mechanical response time of the rotating dipole, this fluctuating torque may be considered as a white Gaussian noise. The resistive torque that was just computed for the QED vacuum can be generalized to the damping by black-body radiation. This damping yields the resistive torque in a Langevin-like theory of the motion of a rotator. The fluctuating Langevin 'torque' balances the resistive torque in such a way that the statistical distribution of the angular speed of the rotating dipole is a Boltzmann distribution at the same temperature as the black-body radiation. To prove that this represents well the torque fluctuations is a rather complicated matter and I refer the interested reader to a forthcoming publication on this.

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REFERENCES

1. J. D. Jackson, *Classical electrodynamics*, 2nd Ed. (Wiley, New York, 1962).
2. R. Lopez-Ruiz and Y. Pomeau, *J. Phys. A* **28**:L255–L259 (1995); Y. Pomeau, *Europhys. Lett.* **27**:377–382 (1994).
3. L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison–Wesley, New York, 1951).
4. Y. Pomeau, *C.R. Physique* **3**:1269–1271 (2002).
5. See for instance problem 3.72 in V. V. Batygin and I. N. Toptygin *Problems in Electrodynamics* (Academic Press, London, 1978).
6. T. H. Boyer, *Ann. Phys.* **56**:474 (1970).
7. D. Roberts and Y. Pomeau *Irreversible Casimir-like Drag in a Bose–Einstein Condensate*, forth coming.
8. An anonymous referee pointed out that the rotating bucket of Newton made an history of its own. Specifically Newton's argument was reconsidered (and seemingly criticized) by E. Mach. It inspired at least in part Einstein when building his theory of General Relativity. This history can be found in the preprint by Lars Rosenberger: "Das Problem der Rotation in der Allgemeinen Relativitaetstheorie" Max Planck Inst. for the History of the Sciences, Berlin, preprint nr. 208.